TOLERABLE PROBABILITY OF LOW LEVEL INTERFERENCE FOR RADIOASTRONOMY.

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ABSTRACT.

We have attempted to estimate the maximum fraction of time loss to low level detrimental interference which can be tolerated for Radioastronomy. These estimates are based on objective criteria, as far as possible at present. For several plausible interference scenarios values of $1-2\%$ are obtained if observational results are to be significant at the 0.13% level (the 3σ criterium).

1) INTRODUCTION.

Interference below the sensitivity limit of a Radioastronomy observation can be tolerated for 100% of the time. Interference much greater than the system sensitivity can in principle be recognised and excised. However in general there will be a loss in measurement precision, due to loss of integration time and to the resulting gaps in UV coverage during aperture synthesis observations for example. Such effects will set an upper limit to the fraction of observing time which can be allowed for such high level interference. Values of a few percent seem to be tolerable in many situations.

Low level interference near the sensitivity limit of the observing system may remain unrecognised and lead to gross errors of interpretation. For example in detection experiments a completely false result may ensue. The history of science in general and astronomy in particular is full of such examples. The reliability of such marginal results is usually assessed by estimating the probability of obtaining the measured value as a result of a chance peak in the instrumental noise. If this probability is less than an agreed threshold, then the measurement is considered reliable. Often a threshold probabilty of 0.13% is taken (corresponding to the 3σ criterion for Gaussian noise with root mean square value of σ), or in more critical situations where the noise statistics are uncertain a probability threshold of 0.00003% (5σ) can be taken. In the present discussion interference can be considered as a special type of noise, non Gaussian, with an unusual probabilty distribution in time and intensity. This non-Gaussian behavior makes the assessement of the probability very difficult and the reliability of a measurement becomes uncertain.

2) STATEMENT OF THE PROBLEM.

The levels of detrimental interference to Radioastronomy, as established by the ITU (in Recommendation ITU-R Rec. 769-1), are defined in terms of the root mean square noise level (σ) of the receiving system after detection and integration. Interference greater than one tenth σ is considered detrimental.

For what fraction of the time, or with what probability, can Radioastronomers tolerate interference which exceeds these levels ? The answer to this question clearly depends on the reliability demanded of the observational results, which in turn depends on the probability threshold for reliable observations which has been adopted. It also depends on the probability distribution of the interfering power.

In the following we try to answer this question for the case of low level interference.

3) PROBABILITY DISTRIBUTIONS.

If the 3σ criterium is chosen, for example, then interference greater than 3σ can not be tolerated for more than 0.13% of the time. This follows from the error function. The corresponding time during which the detrimental level (0.1σ) is exceeded then depends on the probability distribution of interference. In general this is unknown, but on general grounds it may be expected to have an approximately power law variation with intensity over the range of interference levels of interest here.

Then the probability $P(I)$ of exceeding an interference intensity I is

$$P(I) = aI^{-m} \tag{1}$$

where a is a constant of proportionality, and $m \geq 0$ is the power law index which remains to be estimated.

The desired probability $P(0.1\sigma)$ is then given in terms of the probability threshold, for example $P(3\sigma) = 0.13\%$ by

$$P(0.1\sigma) = P(3\sigma)(0.1/3.0)^{-m} \tag{2}$$

4) ESTIMATES OF THE POWER LAW INDEX m.

A) Power law index for a constellation of satellites in non-geostationary orbit.

The estimation of the power law index is simple in this case if it is permitted to assume that the satellites are distributed randomly over the region of the sky surrounding the telescope main beam. On the further assumption that the power flux density at the telescope is also constant, independant of satellite position, then the probability distribution is determined by the sidelobe structure of the telescope beam alone. The intensity of interference received is a function of the position of the satellite within the beam, and its probability is proportional to the element of area. Further assumptions to simplify the analysis are that the radiation pattern $I(r)$ is axially symmetrical of form :-

$$I(r) = Kr^{-n} \tag{3}$$

and that the element of area is rdr (plane geometry), where r is the radial distance from boresight. Differentiation of eq.(3), followed by elimination of the variable r then yields the probability of interference between I and $I + dI$ as

$$dP(I) = \frac{1}{nK} K^{(n+2)/n} I^{(n+2)/n} dI \tag{4}$$

The cumulative distribution, giving the probability of a power greater than I is then

$$P(I) = 0.5K^{(2/n)} I^{-(2/n)} \tag{5}$$

so that the index m of eq.(1) is $2/n$. If spherical geometry is used to derive dP(I) (eq. 4), then an additional factor

$$\sqrt{1 - \left(\frac{K}{I}\right)^{2/n}} \tag{6}$$

appears in eq.(4) with the result that, at low values of I, m increases, and the tolerable probability of interference also increases.

However, such increases are reduced if other factors effecting the interfering power flux density at large angular distances from boresight are taken into account. Atmospheric absorbtion, increased satellite distance and the emission polar diagram of the satellite need to be considered for a more exact analysis.

5) EXAMPLES.

Equations 2 and 5 have been applied to several idealized model antenna patterns which may never-the-less bracket the situation in practice. The tolerable probablility of interference $P(0.1\sigma)$ has been calculated for a probability threshold of 0.13% (the 3σ criterion). The results are listed in the Table 1 below, and are displayed in graphical form in the Figure 1.

Table 1.

Antenna model	n	m	P (0.1σ) %	
Uniform	3.0	0.66	1.25	a
Blocked, -14 dB	3.3	0.60	1.00	b
Blocked, -14 dB	2.7	0.74	1.61	c
ITU Rec. 509	2.5	0.80	1.95	d

a) A circular unblocked uniformly illuminated aperture. This result is based on a one dimensional numerical integration of the analytical expression. It takes the detailed structure of the sidelobes into account.

b) A circular aperture with parabolic taper of -14 dB and a central blockage of fractional diameter 0.1. This result, as for a), is based on a one dimensional numerical calculation which takes the detailed structure of the sidelobes into account.

c) A 2 dimensional numerical simulation for a 30m antenna at 39 GHz. A Gaussian taper of -14 dB is assumed, with blockage (0.077) by a central circular subreflector and its quadrupode support.

d) The model of ITU Rec. 509, for the range of radii between 1 and 48 degrees. This model describes the envelope of the sidelobe maxima and tries to take into account all sources of sidelobe radiation which are not considered by the idealized models of a) and b). It thus yields a conservative upper limit to the tolerable probability of interference.

Figure 1 is a plot of the cumulative probability distributions derived for the models a) b) and c). It shows that for angles beyond the first sidelobe (power less than about -20 dB), the results can be approximated by a power law.

B) A Distribution of ground based transmitters.

A similar, very approximate, treatment can be given for the case of a uniform distribution of ground based transmitters surrounding a radio telescope. Then the coordinate r in equation becomes radial distance on the ground. The problem is simplified if one takes the antenna radiation pattern in the sidelobes illuminating the ground as a constant, as assumed in the ITU Rec. 769 for example. In fact, for the case of a uniform distribution of transmitters the average gain in the direction of the horizon can be used without error. If furthermore, propagation over a plane smooth surface is adopted, the power flux density drops with increasing distance as r^3, as for a dipole source. Then in equation (3), $n = 3$ and in the Table one can find $P(0.1\sigma) = 1.25\%$ for the tolerable probability of interference. Including atmospheric attenuation and diffraction losses over a spherical earth can only increase the effective value for n and decrease $P(0.1\sigma)$.

Hence until the compensating effects of "beyond the horizon propagation" become significant, the value of 1.25% is probably a upper limit for the tolerable probability for detrimental interference.

6) CONCLUSION.

For the idealised cases considered here we see that the tolerable probability for interference at the level detrimental for Radioastronomy lies within the range $1-2\%$, if the results of the observations are to have a statistical reliability of 0.13% (the 3σ criterium).

On the other hand, a probability for tolerable interference of 5% , which is sometimes advocated, would correspond to a reduced statistical reliability of about 0.52% or 1.63σ. This is not usually considered adequate.

It will be interesting to see how closely the statistics of interference agree with the simplified models studied above.